

Chapter 5

5 Marks

PERMUTATIONS AND COMBINATIONS

TOPIC : FACTORIAL

FUNDAMENTAL PRINCIPLES OF COUNTING

PERMUTATIONS

COMBINATIONS

BY : SHIVANI SHARMA



SOMETHING BIG



IS COMING!

14th MAY, AT 8:00 PM



CA BANO YATRA



CA FOUNDATION JUNE'24

FACE-TO-FACE MARATHON

SCAN
TO
REGISTER



JAIPUR

18-19 MAY

UNACADEMY CENTRE

DELHI

21-22 MAY

GHALIB AUDITORIUM,
MANDI HOUSE

KOLKATA

27-28 MAY

UNACADEMY CENTRE



CA Nakul Katheria
Accounting

CA CS Shantam Gupta
Business Law

Shivani Sharma
QA

Akhilesh Daga
Economics

CA Rakesh Kalra
Accounting



Printed Notes/ Question Bank



Exciting Giveaways

SHIVANI SHARMA

QUANTITATIVE APTITUDE ROMANOVA

- **7 years** of experience in teaching **Mathematics**
- **Gold Medalist** in M.Sc. and B.Sc. Maths
- Was **#1 Maths Faculty** in Magnet Brains
- Teaches Maths in **CA foundation** and **K-12**
- Taught **30000+** students with highest score as 100



5 U's to choose Unacademy

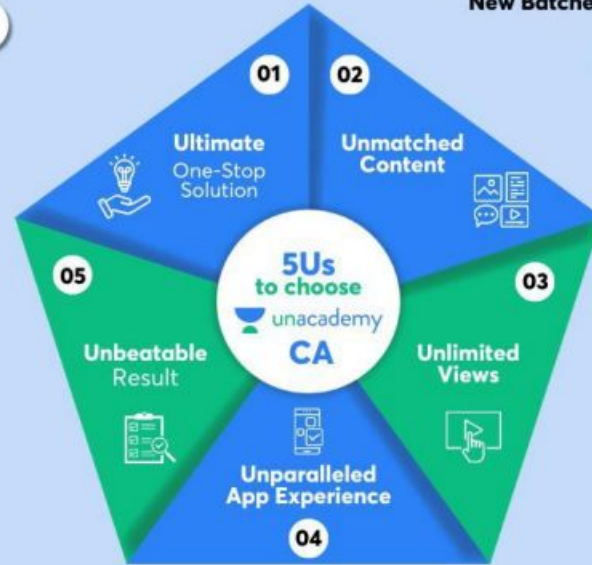
Why join Unacademy for CA Journey?



New Batches Starting This Week

Call to know more

+91 8585858585



Ultimate One-Stop Solution

One Subscription - Get multiple educators per subject, tests & Live doubt-solving



Unmatched Content

Top Educator's content with structured notes, charts, and audio summary



Unlimited Views

Do Multiple Revisions with No Restriction on Views per video



Unparalleled App Experience

Real-time Educator Interaction - Chat, Raise a Hand, Ask a doubt over the call



Unbeatable Results

Consistently delivered India's Best Results in CA



MAHABALI 2.0

CA Foundation Sep 2024 BATCH



25% ~~₹ 9,624~~ ₹6,999

Batch Starts on 27th May



CA Rakesh Kalra
Accounting



CA CS Shantam Gupta
Business Laws



Shivani Sharma
Quantitative Aptitude



CA Aditya Sharma
Business Economics

CA FOUNDATION
DEC'23 RESULTS

357 MARKS

Princi Khemasara
Plus Learner

345 MARKS

Bhumika Goel
Plus Learner

349 MARKS

Yash Wadhwa
Plus Learner

900+ Learners Qualified

130+ Learners scored 300+ marks out of 400

357/400 Top Scorer

CA FOUNDATION

FOR **June 2024**

**25%
OFF**
SALE!

REGULAR

BATCH

QUANTITATIVE APTITUDE

by **Shivani Sharma**



LIVE + RECORDED
WITH UNLIMITED VIEWS



COMPREHENSIVE COVERAGE
OF NEW ADDITIONS

~~₹ 3,499~~

₹ 2,624

Use code- '**SS12**' to avail
discount offer!



PRESENTS

CA PREMIER LEAGUE 2024

ATTEND • PLAY • WIN

Top 3 Winners in every Special CLASS will win Amazon Vouchers

STARTING FROM 26TH MARCH



3rd
₹ 200



2nd
₹ 300



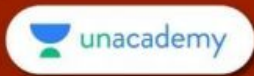
1st
₹ 500



Jab T20 League ka Fever Chadega,
har koi #CAsEbaDaKuchNahi Kahega



Do not miss to attend and win!



Ranker Rewards Program

CA Intermediate/CA Final May'24

Introducing the Ranker Rewards Program to celebrate the success of our **CA Achievers!**



Link in description, update your details!



FACTORIAL

- $0! = 1$
- $1! = 1$
- $2! = 2 \times 1 = 2$
- $3! = 3 \times 2 \times 1 = 6$
- $4! = 4 \times 3 \times 2 \times 1 = 24$
- $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
- $6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$
- $7! = 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 5040$

FACTORIAL

$$n! = n (n-1) (n-2) \dots\dots\dots 3.2.1$$

- $4! = 4 \times 3 \times 2 \times 1$
- $3! = 3 \times 2 \times 1$
- $2! = 2 \times 1$
- $1! = 1$

FACTORIAL

For a natural number n

$$n! = n(n-1)!$$

$$n! = n(n-1)(n-2)!$$

$$n! = n(n-1)(n-2)(n-3)!$$

EXAMPLE

$$5! = 5 \times 4 \times 3 \times 2 \times 1$$

$$5! = 5 \times 4!$$

$$5! = 5 \times 4 \times 3!$$

$$5! = 5 \times 4 \times 3 \times 2!$$



FACTORIAL

RESULT:

$$(n + 1)! - n! = \Rightarrow n.n!$$

Dec 2009

$$(n+1)! = 20(n-1)!$$

a) 6

b) 5

c) 4

d) 10

ANS : C

Nov 2018

The value of N in $\frac{1}{7!} + \frac{1}{8!} = \frac{N}{9!}$ is

(a) 81

(b) 78

(c) 89

(d) 64

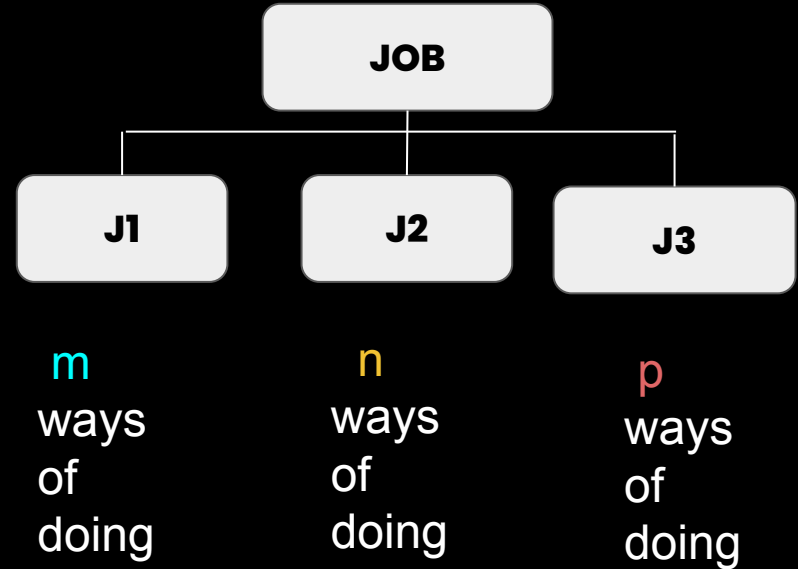
ANS : a

Fundamental Principles of Counting

AND

Multiplication Principle

A job is divided into a number of sub-jobs which are unconnected to each other and the job is said to be performed if each sub-job is performed



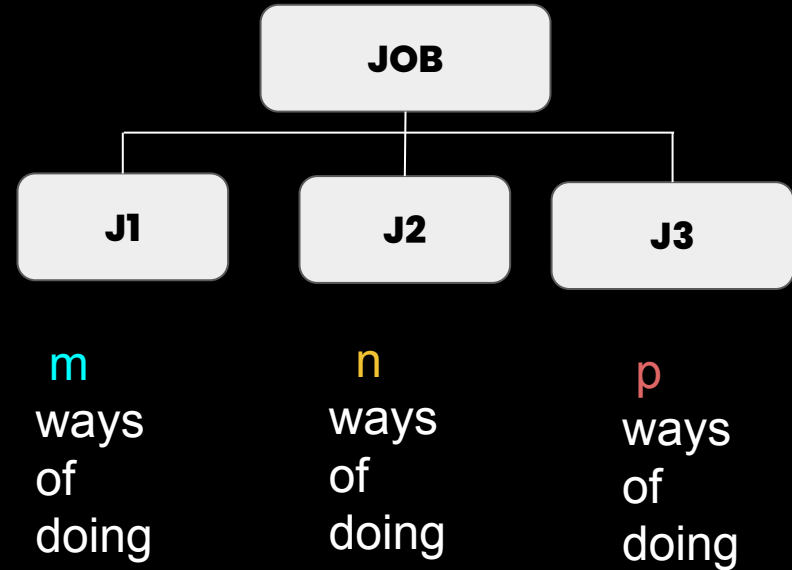
No . of ways of doing the job = $m \times n \times p$

Fundamental Principles of Counting

OR

Addition Principle

There are a number of independent jobs and we have to perform one of them . So the total number of ways of completing any one of the sub -jobs is the sum of the number of ways of completing each sub- jobs



No . of ways of doing the job = $m + n + p$

Jan 2021 ; June 2022

If a man travels from place A to B in 10 ways then by how many ways can he come back by another train?

(a) 94

(b) 110

(c) 90

(d) 99

ANS : C



PERMUTATIONS

- A permutation is an **arrangement in a definite order** of a number of objects taken **some or all at a time** .

PERMUTATIONS



**Arrangement
made with the
letters**

a

b

c

Taking all at a time



- **abc**
- **acb**
- **bac**
- **bca**
- **cab**
- **cba**

PERMUTATIONS



**Arrangement
made with the
letters**

a

b

c

Taking two at a time



- **ab**
- **ba**
- **ac**
- **ca**
- **bc**
- **cb**

PERMUTATIONS

Theorem: The number of permutations of n things when r are chosen at a time

$${}^n P_r = n (n-1)(n-2) \dots (n-r+1)$$

where the product has **exactly r factors**.



RESULTS :

$${}^n P_r = \frac{n!}{(n-r)!}, 0 \leq r \leq n$$

EXAMPLE

- ${}^5 P_3$

- ${}^{10} P_2$



RESULTS :

$${}^n P_r = \frac{n!}{(n-r)!}, 0 \leq r \leq n$$

- ${}^n P_n = n!$

June 2023

If ${}^6P_{2r} = 12 \times {}^6P_r$, then r is equal to

(a) 1

(b) 2

(c) 3

(d) 4

ANS : b

June 2022

The solution for $\frac{n!}{10} = {}^{n-1}P_{n-3}$

(a) 5

(b) 6

(c) 7

(d) 8

ANS : a

Dec 2017

If ${}^n P_{13} : (n+1) P_{12} = 3 : 4$, then n is

(a) 13

(b) 15

(c) 18

(d) 31

ANS : b

July 2021

If ${}^n P_6 = 20 {}^n P_4$, then n is

(a) 5

(b) 3

(c) 9

(d) 8

ANS : c

PERMUTATIONS WHEN ALL THE OBJECTS ARE DISTINCT

- The number of permutations of n **different** objects taken r at a time and objects do not repeat is denoted by ${}^n P_r$

$${}^n P_r = \frac{n!}{(n-r)!}, 0 \leq r \leq n$$

June 2022

How many 4- letter words , with or without meaning , can be formed out of the letters of the word ' LOGARITHMS' if repetition of letters is not allowed.

- a) 7020**
- b) 5040**
- c) 1480**
- d) 2520**

ANS : b

PERMUTATIONS WHEN ALL THE OBJECTS ARE DISTINCT

- The number of permutations of n **different** objects

taken all at a time is denoted by ${}^n P_n$

- ${}^n P_n = n!$

Example How many words , with or without meaning can be formed by using all the letters of the word ' DELHI ' ,using each letter exactly once ?

PERMUTATIONS WHEN ALL THE OBJECTS ARE DISTINCT

- The number of permutations of n different objects taken r at a time, where repetition is allowed, is n^r

Dec 2011

**There are 12 questions are to be answered to be YES or NO .
How many ways can these be answered ?**

- a) 1024**
- b) 2048**
- c) 4096**
- d) None**

ANS : C

PERMUTATIONS WHEN ALL THE OBJECTS ARE **NOT DISTINCT**

The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of second kind, ..., p_k are of k^{th} kind and the rest, if any, are of different kind is

$$\frac{n!}{p_1! p_2! \dots p_k!}$$

June 2017

**Find the number of permutations of the letters of the word
ALLAHABAD.**

- a) 7560**
- b) 3780**
- c) 30240**
- d) 15320**

ANS : a

Exercise 5(A)

Que14. The number of ways the letters of the word 'COMPUTER' can be rearranged is

- (a) 40320
- (b) 40319
- (c) 40318
- (d) None of these

ANS : b

Jan 2021

EVEN / ODD PLACES

In how many different ways can the letters of the word 'DETAIL' be arranged so that the vowels occupy only the odd positions?

- (a) 32**
- (b) 36**
- (c) 48**
- (d) 60**

ANS : b

Dec 2019

**ALTERNATE POSITION
QUESTIONS**

5 boys and 3 girls are to be seated together such that no two girls are together

(a) 14400

(b) 2400

(c) 720

(d) None of these

ANS : a

Dec 2022

**ALTERNATE POSITION
QUESTIONS**

The number of ways 4 boys and 3 girls can be seated in a row so that they are alternate is

- (a) 12**
- (b) 288**
- (c) 144**
- (d) 256**

ANS : C

Exercise 5(D)

ALTERNATE POSITION QUESTIONS

Que.7) The number of words that can be made by rearranging the letters of the word APURNA so that vowels and consonants appear alternate is

- (a) 18
- (b) 35
- (c) 36
- (d) None of these

ANS : C

June 2023

**ALWAYS TOGETHER / NEVER
TOGETHER**

**In how many different ways can the letters of the word
"SOFTWARE " be arranged so that the vowels always come
together**

(a) 720

(b) 144

(c) 2880

(d) 4320

ANS : d

Nov 2018

**ALWAYS TOGETHER / NEVER
TOGETHER**

**The number of words from the letters of the word BHARAT , in
which B and H will never come together is**

(a) 120

(b) 360

(c) 240

(d) None

ANS : C

Dec 2021

**RESTRICTION BASED
QUESTION**

The number of words that can be formed using the letters of " PETROL " such that the words do not have " P" in the first position , is

(a) 720

(b) 120

(c) 600

(d) 540

ANS : C

Exercise 5(B)

RESTRICTION BASED QUESTION

Que15. There are 5 speakers A, B, C, D and E. The number of ways in which A will speak always before B is

- (a) 24
- (b) 4×2
- (c) 5
- (d) None of these

ANS : a

PERMUTATIONS WITH RESTRICTIONS

- **Number of permutations of n distinct objects taken r at a time when a particular object is not taken in any arrangement is**

$${}^{n-1}P_r$$

PERMUTATIONS WITH RESTRICTIONS

- *Number of permutations of r objects out of n distinct objects when **a particular object is always included** in any arrangement*

$$r \cdot {}^{n-1}P_{r-1}$$

June 2011

**RESTRICTION BASED
QUESTION**

Find the number of arrangements of 5 things taken out of 12 things , in which one particular thing must always be included

- (a) 39,000**
- (b) 37,600**
- (c) 39,600**
- (d) 36,000**

ANS : C

Dec 2022

NUMBER FORMATION

How many 3 digit odd numbers can be formed using the digits 5, 6, 7, 8, 9, if the digits can be repeated ?

(a) 55

(b) 75

(c) 65

(d) 85

ANS : b

July 2021

NUMBER FORMATION

How many numbers of 7 digit numbers which can formed from the digits 3, 4, 5, 6, 7, 8, 9 no digits being repeated are not divisible by 5

(a) 4320

(b) 4690

(c) 3900

(d) 3890

ANS : a

Dec 2019

NUMBER FORMATION

How many numbers divisible by 5 of 6 digit can be made from the digit 2 , 3, 4, 5, 6, 7

(a) 120

(b) 600

(c) 240

(d) None

ANS : a

June 2015

NUMBER FORMATION

The number of 4 digit numbers that can be formed from seven digits 1, 2, 3, 5, 7, 8, 9 such that no digit being repeated in any number, which are greater than 3000

(a) 120

(b) 600

(c) 240

(d) None

ANS : b

June 2010

NUMBER FORMATION

How many numbers not exceeding 1000 can be made from the digits 1, 2, 3, 4, 5, 6, 7, 8, 9 if repetition is not allowed

(a) 364

(b) 585

(c) 728

(d) 819

ANS : b

Jan 2021

NUMBER FORMATION

How many odd numbers of four digit can be formed with digit 0 , 1, 2, 3, 4, 7, 8 ?

(a) 150

(b) 300

(c) 120

(d) 210

ANS : b

Exercise 5(B)

NUMBER FORMATION

Que10. The number of numbers lying between 10 and 1000 can be formed with the digits 2, 3, 4, 0, 8, 9 is

- (a) 124
- (b) 120
- (c) 125
- (d) None of these

ANS : C

Exercise 5(A)

NUMBER FORMATION

Que19. The sum of all 4 digit number containing the digits 2, 4, 6, 8, without repetitions is

(a) 1,33,330

(b) 1,22,220

(c) 2,13,330

(d) 1,33,320

ANS : d

Step 1 : Find all 4 digits number

Step 2 : Repetition of each digits

$$\frac{\text{Total numbers}}{\text{4 digit number}}$$

Step 3 : Sum of given digits

Step 4 : step 2 x step 3

Step 5 : step 4 x 1111 (as 4 digit number)

CIRCULAR PERMUTATIONS

- **The number of circular permutations of n different things chosen all at a time is**

$$(n-1)!$$

- **The number of ways of arranging n persons along a round table so that no person has the same two neighbours is $= \frac{1}{2}(n-1)$**

- ***The number of necklaces formed with n beads of different* $= \frac{1}{2}(n-1)$**

EXERCISE 5(B)

CIRCULAR

Que2. The number of ways in which 7 boys sit in a round table so that two particular boys may sit together is

- (a) 240
- (b) 200
- (c) 120
- (d) None of these

ANS : a

July 2021

CIRCULAR

The number of ways in 5 boys and 5 girls can be seated at a round table , so no two boys are adjacent

(a) 2550

(b) 2880

(c) 625

(d) 2476

ANS : b

COMBINATIONS

X, Y, Z

TEAM OF 2 PLAYERS IS TO BE FORMED

X, Y

Y, Z

X, Z



COMBINATIONS

- In a **combination only selection is made** whereas in a **permutation not only a selection is made but also an arrangement in a definite order is considered.**
- In a combination , the ordering of the selected objects is immaterial whereas in a permutation , the ordering is essential . **For example :**
A, B and B, A are same as combinations but different as permutations

COMBINATIONS

- ${}^n C_r = \frac{n!}{r!(n-r)!}, 0 \leq r \leq n.$

EVALUATE:

$${}^{10} C_3$$

- ${}^n C_0 = 1$

- ${}^n C_n = 1$

EVALUATE:

- ${}^n C_r = {}^n C_{n-r}$

$${}^{100} C_{97}$$

COMBINATIONS

$${}^n P_r = {}^n C_r r!,$$

$${}^n C_a = {}^n C_b \Rightarrow a = b \text{ or } n = a + b$$

$${}^n C_r + {}^n C_{r-1} = {}^{n+1} C_r$$

June 2019

If ${}^{11}C_x = {}^{11}C_{2x-4}$ and $x \neq 4$, then the value of 7C_x is:

(a) 20

(b) 21

(c) 22

(d) 23

ANS : b

June 2018

If ${}^{12}C_3 + 2 \cdot {}^{12}C_4 + {}^{12}C_5 = {}^{14}C_x$, The value of x

- a) 3 or 5
- b) 5 or 9
- c) 7 or 1
- d) 9 or 12

ANS : b

June 2014

If ${}^{1000}C_{98} = {}^{999}C_{97} + {}^x C_{901}$, then the value of x will be:

- (a) 999
- (b) 998
- (c) 997
- (d) None of these.

ANS : a



SELECT One or More

Combinations of n different things taking **one or more** out of n things
at a time

$$2^n - 1$$

Dec 2015

**A question paper consist 10 questions , 6 in math and 4 in stats .
Find out number of ways to solve question paper if atleast one
question is to be attempted from each section**

(a) 1024

(b) 950

(c) 945

(d) 1022

ANS : c

NUMBER OF FACTORS OF A NUMBER

- Write the given number say N in the form $N = p^a \cdot q^b \cdot r^c$

where p, q, r are prime factors of the number N .

- Number of factors $N = (a + 1) (b + 1) (c + 1)$

Exercise 5(D)

Choose the most appropriate option (a), (b), (c) or (d)

Que.12) The number of different factors the number 75600 has is

- (a) 120
- (b) 121
- (c) 119
- (d) None of these

ANS : c

IMPORTANT FORMULAE

Number of straight lines with the given n points

$${}^n C_2$$

Number of Straight lines with the given n
points where m points are collinear

$${}^n C_2 - {}^m C_2 + 1$$

IMPORTANT FORMULAE

Number of triangles with the given n points

$${}^n C_3$$

Number of triangles with the given n points where
 m points are collinear

$${}^n C_3 - m C_3$$

IMPORTANT FORMULAE

Number of parallelogram with the given one set of m parallel lines and another set of n parallel lines

$${}^m C_2 \times {}^n C_2$$

Number of Diagonals with n sides

$${}^n C_2 - n$$

The maximum number of points of intersection of n circles will be

$$n(n-1)$$

Dec 2022

There are 20 points in a plane area. How many triangles can be formed by these points if 5 points are collinear?

(a) 550

(b) 560

(c) 1130

(d) 1140

ANS : c

June 2013

A polygon has 44 diagonals then the number of its sides are:

(a) 8

(b) 9

(c) 10

(d) 11

ANS : d

Dec 2010

How many ways a team of 11 players can be made out of 15 players if one particular player is not to be selected in the team.

(a) 364

(b) 728

(c) 1,001

(d) 1,234

ANS : a

June 2013

Number of ways of shaking hands in a group of 10 persons shaking hands to each other are:

(a) 45

(b) 54

(c) 90

(d) 10

ANS : a

June 2015

A person has ten friends of whom six are relatives. If he invites five guests such that three of them are his relatives, then the total number of ways in which he can invite them are:

(a) 30

(b) 60

(c) 120

(d) 75

ANS : c

Dec 2015

There are 6 men and 4 women in a group, then the number of ways in which a committee of 5 persons can be formed of them, if the committee is to include at least 2 women are:

- (a) 180**
- (b) 186**
- (c) 120**
- (d) 105**

ANS : b

Dec 2020

A fruity basket contains 7 apples, 6 bananas, and 4 mangoes. How many selections of 3 fruits can be made so that all 3 are apples?

- (a) 35 ways**
- (b) 120 ways**
- (c) 165 ways**
- (d) 70 ways**

ANS : a

Jan 2021

A business houses wishes to simultaneously elevate two of its six branch heads. In how many ways can these elevations take place?

(a) 12

(b) 3

(c) 6

(d) 15

ANS : d

Jan 2021

'n' locks and 'n' corresponding keys are available but the actual combination is not known. The maximum number of trials that are needed to assigns the keys to the corresponding locks is:

(a) ${}^{(n-1)}C_2$

(b) ${}^{(n+1)}C_2$

(c) $\sum_{k=2}^n (k-1)$

(d) $\sum_{k=2}^n k$

ANS : c

June 2023

A committee of 3 women and 4 men is to be formed out of 8 women and 7 men. Mrs. Kajal refuses to serve in a committee in which Mr.Yash is a member. The number of such committee can be.

(a) 1530

(b) 1500

(c) 1520

(d) 1540

ANS : d

Exercise 5(D)

Choose the most appropriate option (a), (b), (c) or (d)

Que.2) The ways of selecting 4 letters from the word 'EXAMINATION' is

- (a) 136
- (b) 130
- (c) 125
- (d) None of these

ANS : a

Exercise 5(D)

Choose the most appropriate option (a), (b), (c) or (d)

Que.13) The number of 4 digit numbers formed with the digits 1, 1, 2, 2, 3, 4 is

- (a) 100
- (b) 101
- (c) 201
- (d) None of these

ANS : d

Exercise 5(D)

Choose the most appropriate option (a), (b), (c) or (d)

Que.3) The number of different words that can be formed with 12 consonants and 5 vowels by taking 4 consonants and 3 vowels in each word is

(a) ${}^{12}C_4 \times {}^5C_3$

(b) ${}^{17}C_7$

(c) $4950 \times 7!$

(d) None of these

ANS : c

Exercise 5(c)

Choose the most appropriate option (a), (b), (c) or (d)

Que.21) The Supreme Court has given a 6 to 3 decision upholding a lower court; the number of ways it can give a majority decision reversing the lower court is

- (a) 256
- (b) 276
- (c) 245
- (d) 226

ANS : a